(54) Title: PHASE DETERMINATION OF A RADIATION WAVEFIELD
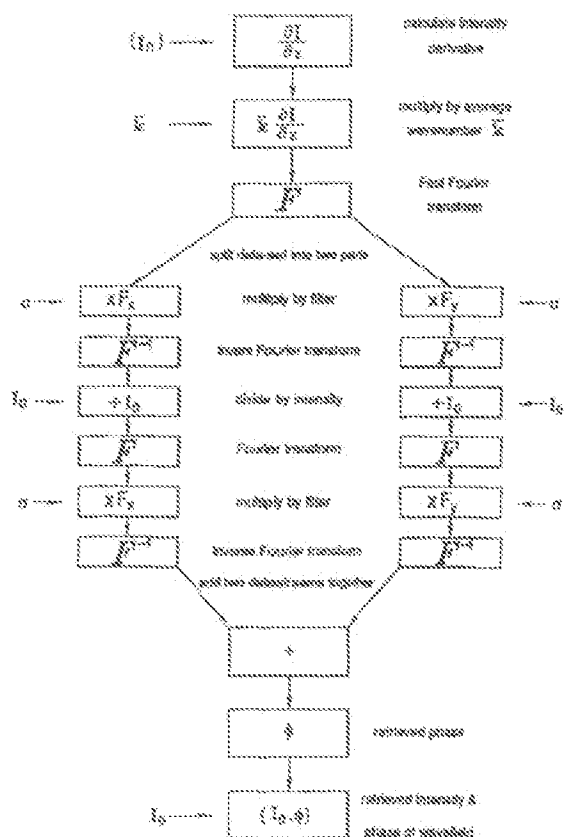
(57) Abstract: The phase of radiation wavefield is retrieved by solving the transport of intensity equation. The rate of change of intensity, orthogonal to a surface extending across the wavefield, is first determined (eg. by measuring intensities at two separated planes). This rate of change is subjected to the computational process of taking an integral transform, multiplying by a filter corresponding to the inversion of a differential operator, and taking the inverse integral transform. The result is multiplied by a function of the intensity over the surface, and subjected to a similar computational process, to obtain a measure of the phase over the surface. The filters have a form based on details of the optical system used to acquire the intensity data, such as numerical apertures and spatial frequencies.

Eurasian patent (AM, AZ, BY, KG, KZ, MD, RU, TJ, TM), European patent (AT, BE, BG, CH, CY, CZ, DE, DK, EE, ES, FI, FR, GB, GR, IE, IT, LU, MC, NL, PT, SE, SK, TR), OAPI patent (BF, BJ, CF, CG, CI, CM, GA, GN, GQ, GW, ML, MR, NE, SN, TD, TG).

For two-letter codes and other abbreviations, refer to the "Guidance Notes on Codes and Abbreviations" appearing at the beginning of each regular issue of the PCT Gazette.

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PHASE DETERMINATION OF A RADIATION WAVEFIELD

Field of the Invention
This invention relates to the determination of phase of a
5   radiation wavefield to enable phase images of objects to
be produced. This invention is an improvement to the
method for determining the phase of a radiation wavefield
disclosed in International Patent Application No.
PCT/AU99/00949. The contents of this International
10   application are incorporated into this specification by
this reference. As in the above International
application, the term radiation wavefield is intended to
include all forms of radiation that propagate in a wave-
like manner, including but not limited to x-rays, visible
15   light and electrons.

Background of the Invention
The method disclosed in the above International
application enables phase data relating to the radiation
20   wavefield to be determined from which a phase image can be
constructed. The determination of the phase data relating
to the radiation wavefield also enables other image
modalities to be produced such as DIC, Zernike, Hoffman
Contrast Images and Darkfield images. Phase images are an
25   important tool to microscopists because they enable detail
of some objects to be ascertained which are not available
in conventional contrast images. In particular, fine edge
detail in transparent structures such as cells and other
biological samples may be more visible when phase images
30   are taken rather than conventional contrast images.
Furthermore, the phase data, as described in Australian
Provisional Application No. PR5928, also enables the other
modalities referred to above to be determined in software
rather than by optics in a microscope.

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The method for determining the phase of a radiation
wavefield according to the above International application

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requires the so-called transport of intensity equation to
be solved. This is achieved by producing a representative
measure of the rate of change of intensity of the
radiation wavefield over a selected surface extending
generally across the wavefield, producing a representative
measure of intensity of the radiation wavefield over the
selected surface, transforming the measure of rate of
change to produce a first integral transform
representation and applying to that representation a first
filter corresponding to the inversion of a first
differential operator. The inverse of the first integral
transform is then applied to the first modified integral
transform representation and a correction based on the
measure of intensity over the selected surface to the
untransformed representation is then applied. The correct
untransformed representation is then transformed to
produce a second integral transform representation and a
second filter corresponding to the inversion of a second
differential operator is applied to that representation.
An inverse of the second integral transform is then
applied to the second modified integral transform
representation to produce a measure of phase of the
radiation wavefield across a selected plane.

The form of the first and second differential operators
according to the teachings of the above International
application has the form

$$\partial_x^{-1} \nabla_\perp^2$$

in real space and incorporates a term with the form

$$\frac{k}{k^2 + \alpha^2}$$

in its Fourier representation.

In the above equation, k is the spatial frequency and α is
an arbitrary constant which is included so that when k
approaches 0, the operator does not diverge because of the
inclusion of the factor α². The inversion of a first
5   differential operator which is frequency dependent will,
if the factor α is not included, tend to highlight or
exaggerate low frequencies relative to high frequencies
thereby masking the high frequencies which will cause a
degradation in sharpness of the image obtained from the
10 phase data over what may have otherwise been achieved.
Since fine edge detail in transparent samples is likely to
be high frequency in nature, the minimisation of the high
frequencies therefore results in a loss of information in
the image produced from the phase data and therefore lack
15 of clarity in the compiled image.

In order to prevent the exaggerating of the low
frequencies relative to the high frequencies, the factor α
in the above equation decreases the value of the above
20 operator when frequency is small so that the low frequency
signals do not swamp or become over-exaggerated compared
to the high frequency signals which are likely to contain
most of the information of interest.

25 Summary of the Invention
The objection of the present invention is to provide
further improvements to the method and, in particular, to
the form of the differential operator used in the method.

30 The invention may be said to reside in a method of
quantitative determination of a phase of a radiation
wavefield including the steps of:
          (a)   producing a representative measure of the
rate of change of intensity of said radiation wave field
35 over a selected surface extending generally across the
wave field;
          (b)   producing a representative measure of

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intensity of said radiation wave field over said selected
surface;

(c)   transforming said measure of rate of change
of intensity to produce a first integral transform
representation and applying to said first integral
transform representation a first filter corresponding to
the inversion of a first differential operator reflected
in said measure of rate of change of intensity to produce
a first modified integral transform representation;

(d)   applying an inverse of said first integral
transform to said first modified integral transform
representation to produce an untransformed representation;

(e)   applying a correction based on said measure
of intensity over said selected surface to said
untransformed representation;

(f)   transforming the corrected untransformed
representation to produce a second integral transform
representation and applying to said second integral
transform representation a second filter corresponding to
the inversion of a second differential operator reflected
in the corrected untransformed representation to produce a
second modified integral transform representation;

(g)   applying an inverse of said second integral
transform to said second modified integral transform
representation to produce a measure of phase of said
radiation wave field across said selected plane; and

(h)   wherein at least one of the first or second
differential operator has a form based on an optical
system used to acquire the radiation for producing the
representative measure of the rate of change of intensity
of the radiation wavefield over the selected surface
extending generally across the wavefield.

Because the form of the differential operator is based on
the actual optical system, the form of the differential
operator is improved and therefore provides better results
because the operator is based on the optical system.

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It should be understood that the reference to optical system in this specification includes conventional glass or plastic lenses and like imaging elements, as well as magnetic fields or electric fields which are used to condition radiation in the form of electrons in, for example, electron microscopes.

Preferably both the first and second differential operators have a form based on the optical system.

Preferably the first and second integral transforms are produced using a Fourier transform.

In one embodiment of the invention the differential operators have the form:

$$\frac{\sqrt{T_p}}{T_p + \alpha^2}$$

where,

$$T_p(\rho) = 2\pi i \, \& \int \eta T_p^{(1)}(\rho,\eta) d\eta$$

and

$$T_p^{(1)}(\rho,\eta) = \frac{i}{2\pi \rho} \left\{ \left[\tfrac{1}{4}\rho_{obj}^2\left(\xi^2+1\right) - \tfrac{1}{4}\rho^2 - \left(\frac{\eta}{\lambda\rho}\right)^2 - \left|\frac{\eta}{\lambda} - \tfrac{1}{4}\rho_{obj}^2\left(\xi^2-1\right)\right|\right]^{\frac{1}{2}} - \left[\tfrac{1}{4}\rho_{obj}^2\left(\xi^2+1\right) - \tfrac{1}{4}\rho^2 - \left(\frac{\eta}{\lambda\rho}\right)^2 - \left|\frac{\eta}{\lambda} + \tfrac{1}{4}\rho_{obj}^2\left(\xi^2-1\right)\right|\right]^{\frac{1}{2}} \right\}$$

and wherein ρ is radial lateral spatial frequency, the η longitudinal spatial frequency and $\&$ is the defocus distance in the plane of the object. Also

$$\xi = \frac{NA_{condenser}}{NA_{objective}}$$

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where $NA_{condensor}$ and $NA_{objective}$ are respectively the numerical
aperture of the condensor and the objective (These are
settings and dimensions on the microscope). ρ_{obj} is the
maximum spatial frequency accepted by the objective.

The invention may also be said to reside in a computer
program for quantitative determination of the phase of a
radiation wavefield including code to perform the method
steps described above.

The invention may also be said to reside in an apparatus
for phase amplitude imaging of an object including:

     a radiation wavefield source to irradiate the
object;

     an imaging system to focus radiation from the
object to an imaging surface extending across the
wavefield propagating from the object;

     means to produce a representative measure of
radiation intensity over the imaging surface; and

     processing means to:

     (i)   produce a representative measure of
intensity and a representative measure of rate of change
of intensity in the direction of radiation propagation
over a selected surface extending across the wave field
from representative measures of radiation intensity over
said image surface at said first focus and said second
focus;

     (ii)   transform said measure of rate of change
of intensity to produce a first integral transform
representation;

     (iii)   apply to said first integral transform
representation a first filter corresponding to the
inversion of a first differential operator reflected in
said measure of rate of change of intensity to produce a
first modified integral transform representation;

     (iv)   apply an inverse of said first integral
transform to said first modified integral transform

representation to produce an untransformed representation;

(v)   apply a correction based on said measure of intensity over said selected surface to said untransformed representation;

(vi)   transform the corrected untransformed representation to produce a second integral transform representation;

(vii)   apply to said second integral transform representation a second filter corresponding to the inversion of a second differential operator reflected in the corrected untransformed representation to produce a second modified integral transform representation; and

(viii) apply an inverse of said second integral transform to said second modified integral transform representation to produce a measure of phase of said radiation wave field across said selected plane; and

(ix)   wherein at least one of the first or second differential operators has a form based on the imaging system to focus the radiation from the object to the imaging surface.

Preferably the differential operator, in Fourier space, is

$$\frac{\sqrt{T_p}}{T_p + \alpha^2}$$

where,

$$T_p(\rho) = 2\pi i \lambda \int \eta T_p^{(1)}(\rho, \eta) d\eta$$

and

$$T_p^{(1)}(\rho, \eta) = \frac{i}{2\pi\rho} \left\{ \left[\tfrac{1}{4}\rho_{obj}^2(\xi^2+1) - \tfrac{1}{4}\rho^2 - \left(\frac{\eta}{\lambda\rho}\right)^2 - \left|\frac{\eta}{\lambda} - \tfrac{1}{4}\rho_{obj}^2(\xi^2-1)\right| \right]^{1/2} - \left[\tfrac{1}{4}\rho_{obj}^2(\xi^2+1) - \tfrac{1}{4}\rho^2 - \left(\frac{\eta}{\lambda\rho}\right)^2 - \left|\frac{\eta}{\lambda} + \tfrac{1}{4}\rho_{obj}^2(\xi^2-1)\right| \right]^{1/2} \right\}$$

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Preferably both the first and second differential operators have the above form.

Brief Description of the Drawings

A preferred embodiment of the invention will be described, by way of example, with reference to the accompanying drawings in which:

Figure 1A is a schematic illustration of an arrangement for determination of phase where an object is illuminated with plane wave radiation;

Figure 1B is an illustration similar to Figure 1A, but with the object illuminated with point source radiation;

Figure 2 is a flow chart showing an implementation of the method of phase determination in accordance with an embodiment of this invention;

Figure 3 is a schematic illustration of an arrangement for phase amplitude microscopy using the method of the preferred embodiment of the invention;

Figure 4 is a schematic drawing of an exemplary system for quantitative phase amplitude microscopy according to the preferred embodiment of the invention; and

Figure 5 is a flow chart according to the preferred embodiment.

Figures 1(a) and (b) show a schematic arrangement for phase determination in accordance with this invention where an object is illuminated by plane-wave radiation 2 or point source radiation 2 to produce transmitted beams 3.

At each point in space, an optical beam possesses two properties: intensity and phase. Intensity is a measure of the amount of energy flowing through each point, while phase gives a measure of the direction of the energy flow.

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Intensity may be measured directly, for example by
recording an image on film. Phase is typically measured
using interference with a "reference beam". In contrast
5   the present method gives a non-interferometric method for
measuring phase.

Intensity can be measured over two parallel planes A, B
extending across the direction of propagation of the wave
10 field on the side remote from the incident radiation.

The present invention determines phase by providing a
solution to the transport-of-intensity equation:

15   (1) $\nabla_\perp \bullet (I \nabla_\perp \phi) = -k \frac{\partial I}{\partial z}$

where I is the intensity in the plane, the gradient
operator in the plane is denoted $\nabla_\perp$, k is the wave number
of the radiation, and $\partial I / \partial z$ is the intensity derivative or
20 rate of change of intensity. Note that $\partial I / \partial z$ is estimated
from the difference of the measurements in the planes A &
B shown in Figure 1, while the intensity I is given by the
average of the measurements.

25 In order to obtain a solution to equation 1 the function A
is first defined as:
          (2) $\nabla_\perp A = I \nabla_\perp \phi$.

Thus equation (1) becomes:
30
          (3) $\nabla_\perp \bullet (\nabla_\perp A) = -k \partial_z I$.

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Making use of a standard identity $\nabla_\perp \cdot \nabla_\perp = \nabla_\perp^2$, this may be written:

5   (4) $\nabla_\perp^2 A = -k\partial_z I$

where $\nabla_\perp^2$ denotes the two-dimensional Laplacian acting over the surface of the image. This equation has the following symbolic solution:

10   (5) $A = -k\nabla_\perp^{-2}\partial_z I$.

If the gradient operator $\nabla_\perp$ is applied to both sides of this equation, it becomes:

15   (6) $\nabla_\perp A = -k\nabla_\perp\nabla_\perp^{-2}\partial_z I$.

The defining equation (2) for the function A allows (6) to be transformed into:

20   (7) $I\nabla_\perp\phi = -k\nabla_\perp\nabla_\perp^{-2}\partial_z I$.

Dividing both sides by I then yields:

25   (8) $\nabla_\perp\phi = -kI^{-1}\nabla_\perp\nabla_\perp^{-2}\partial_z I$.

Taking the two dimensional divergence $\nabla_\perp \cdot$ of both sides of (8), and again making use of the standard identity $\nabla_\perp \cdot \nabla_\perp = \nabla_\perp^2$, then (8) becomes:

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$$(9) \qquad \nabla_\perp^2 \phi = -k \nabla_\perp \bullet \left[I^{-1} \nabla_\perp \nabla_\perp^{-2} \partial_z I \right].$$

This equation has the following symbolic solution:

$$(10) \qquad \phi = -k \nabla_\perp \left(\nabla_\perp \bullet \left[I^{-1} \nabla_\perp \nabla_\perp^{-2} \partial_z I \right] \right).$$

In order to implement a practical solution to equation (10), the following formulae are required. A suitably-well-behaved function f(x,y) may be written in the form of a two-dimensional Fourier integral:

$$(11) \qquad f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{f}(k_x, k_y) e^{i(k_x x + k_y y)} dk_x dk_y.$$

The function $\hat{f}(k_x, k_y)$ is called the "Fourier transform" of f(x,y).

The x derivative of (11) yields:

$$(12) \qquad \frac{\partial}{\partial x} f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ik_x \hat{f}(k_x, k_y) \right] e^{i(k_x x + k_y y)} dk_x dk_y.$$

Hence the Fourier transform of $\frac{\partial}{\partial x} f(x,y)$ is equal to the Fourier transform of f(x,y) multiplied by ik_x. Stated differently, $\frac{\partial}{\partial x} = iF^{-1} k_x F$, where F denotes Fourier transformation and F^{-1} denotes inverse Fourier transformation. Similar considerations apply to $\frac{\partial}{\partial y} f(x,y)$.

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If the Laplacian $\nabla_\perp^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ of (11) is obtained and

similar reasoning applied, it follows that $\nabla_\perp^2 = -F^{-1}k_r^2 F$,

where $k_r^2 = k_x^2 + k_y^2$. Thus:

$$(13) \quad \nabla_\perp^2 = -F^{-1}k_r^2 F, \quad \frac{\partial}{\partial x} = iF^{-1}k_x F, \quad \frac{\partial}{\partial y} = iF^{-1}k_y F.$$

Here, F denotes Fourier transformation, F^{-1} denotes inverse
Fourier transformation, (k_x, k_y) are the Fourier variables
conjugate to (x,y), and

$$k_r^2 = k_x^2 + k_y^2.$$

Equations (13) can be used to rewrite equation (10) in the
form

$$(14)$$

$$\phi = \phi^{(x)} + \phi^{(y)}, \begin{cases} \phi^{(x)} = F^{-1}k_r^{-2}k_x F I^{-1} F^{-1}k_x k_r^{-2}F\left[k_x\frac{\partial I}{\partial x}\right] \\[3mm] \phi^{(y)} = F^{-1}k_r^{-2}k_y F I^{-1} F^{-1}k_y k_r^{-2}F\left[k_y\frac{\partial I}{\partial y}\right] \end{cases}$$

In practice division by intensity is only performed if
that intensity is greater than a certain threshold value
(eg. 0.1% of the maximum value).

Division by k_r does not take place at the point $k_r = 0$ of
Fourier space;

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instead multiplication by zero takes place at this point. This amounts to taking the Cauchy principal value of the integral operator $\nabla_{\perp}^{-2}$.

5   In order to quantitatively measure the phase of object it is necessary to incorporate some physical constants into the phase recovery algorithm given in Equation (14) relating to the experimental setup in use to quantify the variables k_x, k_y. This can be done by rewriting equation

10 (14) in the following form suitable for implementation using a fast Fourier transform:

$$\phi = \phi_x + \phi_y, \begin{cases} \phi_x = -\dfrac{2\pi}{\lambda \delta\!z}\dfrac{1}{(N\Delta x)^2}\mathsf{F}^{-1}\dfrac{i}{i^2+j^2}\mathsf{F}\dfrac{1}{I(x,y)}\mathsf{F}^{-1}\dfrac{i}{i^2+j^2}\mathsf{F}\{I_+ - I_-\} \\[2mm] \phi_y = -\dfrac{2\pi}{\lambda \delta\!z}\dfrac{1}{(N\Delta x)^2}\mathsf{F}^{-1}\dfrac{j}{i^2+j^2}\mathsf{F}\dfrac{1}{I(x,y)}\mathsf{F}^{-1}\dfrac{j}{i^2+j^2}\mathsf{F}\{I_+ - I_-\} \end{cases}$$

15 where $i, j \in \left[\dfrac{-N}{2}, \dfrac{N}{2}\right]$ index the frequent components of $F\{I_+ - I_-\}$

where the intensity derivative $\partial_z I(x,y)$ is obtained by subtracting two images I_+ and I_- separated by a distance $\delta\!z$, i and j are the pixel numbers on the image, and using the fact that the Fourier space step size is given by

20

$$\Delta k = \frac{1}{N\Delta x}$$

where the image is an N x N array of pixels of size Δx. Thus in addition to measuring the three intensity

25 distributions it is necessary to know the pixel size Δx, defocus distance $\delta\!z$ and wavelength λ in order to make a quantitative phase measurement. All of these quantities

can be readily determined: the pixel size can be
determined directly for example from the CCD detector
geometry (in the case of direct imaging), or by existing
techniques for calibrating the transverse distance scales

5   (in the case of an imaging system), the defocus distance
can be measured directly, and the spectral distribution of
the illumination can be determined either by
monochromating the incident field or by analysing the
spectral distribution of the radiation using existing

10   spectroscopic methods.

An example of the phase-retrieval method implementing the
solution of equation (14) can be represented by the
flowchart shown in Figure 2. As shown in Figure 2 the

15   quantitative determination of phase of a radiation wave
field proceeds from a set of intensity measurements $\{I_n\}$
over the two spaced apart planes A and B. A measurement
of central intensity $I(x,y)$ in a selected plane parallel
to and midway between the planes A and B is also obtained.

20   The intensity measurements are performed over a defined
array on each of the two planes A and B and the respective
values subtracted to produce a measure of the intensity
derivative. This value is multiplied by the negative of
the average wave number. The data are split into two

25   component sets and a fast Fourier transform is performed
to produce the respective x and y components in the
Fourier domain. A filter is then applied to the Fourier
domain representations to correspond to the inversion of a
differential operator reflected in the untransformed

30   representation. An inverse Fourier transform is then
performed on each of the x and y components to produce a
spatial domain value from which the differential operator
has been removed. A division by the central intensity

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I(x,y) obtained by averaging the intensity measurements
over planes A and B is then performed if the intensity
level is above a selected threshold level. The resultant
data is again Fourier transformed and multiplied by the
5   same filter to again correspond to the inversion of a
differential operator reflected in the untransformed data.
The resultant components are again inverse Fourier
transformed and summed to provide a retrieved phase
measurement.

10

It will be apparent that in general the method according
to this invention can proceed from any suitable
representative determination of intensity derivative or
rate of change of intensity over a selected surface
15   extending across the propagation direction and the
intensity over that same surface. As will be explained in
various examples these data can be obtained in a variety
of ways and the method implemented to yield phase of the
radiation wave field.

20

Rewriting equation (14) with:

$$\Omega_x(k_x, k_y, \alpha) = k_x k_r^{-2}$$

25

$$\Omega_y(k_x, k_y, \alpha) = k_y k_r^{-2}$$

$$\phi(x,y) = \phi^{(x)}(x,y) + \phi^{(y)}(x,y),$$

gives

30

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(15)

$$\phi^{(x)}(x,y)=\mathbf{F}^{-1}\Omega_x(k_x,k_y,\alpha)\mathbf{F}\frac{1}{I(x,y)}\mathbf{F}^{-1}\Omega_x(k_x,k_y,\alpha)\mathbf{F}\left[\overline{k}\frac{\partial I}{\partial z}\right]$$

$$\phi^{(y)}(x,y)=\mathbf{F}^{-1}\Omega_y(k_x,k_y,\alpha)\mathbf{F}\frac{1}{I(x,y)}\mathbf{F}^{-1}\Omega_y(k_x,k_y,\alpha)\mathbf{F}\left[\overline{k}\frac{\partial I}{\partial z}\right]$$

where:

$\phi(x,y)$ denotes the recovered phase,

$\mathbf{F}$ denotes Fourier transformation, and $\mathbf{F}^{-1}$ denotes inverse Fourier transformation,

$I(x,y)$ is the intensity distribution over the plane of interest,

(x,y) are Cartesian coordinates over the plane of interest,

(k_x,k_y) are the Fourier variables conjugate to (x,y)

$\overline{k}=2\pi/\overline{\lambda}$ is the average wavenumber of the radiation,

$\overline{\lambda}$ is the average wavelength of the radiation,

$\partial I/\partial z$ is the estimate for the longitudinal intensity derivative,

α is the regularization parameter used to stabilize the algorithm when noise is present.

As given above, the solution to the transport of intensity equation (1) assumes a perfect imaging system. That is, there are no "aberrations" present in the optical system used to obtain the intensity data which is fed into the algorithm. Of course, no imaging system is perfect. The imperfections present in an imaging system may be quantified by a set of numbers:

(16) $A_1, A_2, A_3, ...$

which are termed aberration coefficients.

If intensity data were taken on an imperfect instrument whose imperfections were characterized by a certain set of known aberration coefficients $A_1, A_2, A_3, ...$, it would be desirable if the filters $\Omega_x(k_x k_y, \alpha)$ and $\Omega_y(k_x k_y, \alpha)$ present in (15) could be replaced by modified filters which explicitly depend upon the aberration coefficients:

(17) $\tilde{\Omega}_x(k_x k_y, \alpha, A_1, A_2, A_3, ...)$ and $\tilde{\Omega}_y(k_x k_y, \alpha, A_1, A_2, A_3, ...)$

This would allow the imperfections of the imaging system to be explicitly taken into account, leading to quantitatively correct phase retrieval using aberrated imaging systems. For the special case of a non-absorbing phase object in a radiation wave field of uniform intensity with weak (i.e. much less than 2π radians) phase variations the appropriate modified filters lead to the following functional form for the phase-retrieval algorithm:

(18) $\phi(x,y) = \mathbf{F}^{-1} \left\{ \dfrac{\mathbf{F} \{ I_{aberrated}(x,y) - 1 \}}{-2\pi \& \lambda (k_x^2 + k_y^2) - 2 \sum_m \sum_n A_{mn} k_x^m k_y^n} \right\}$,

where:

$I_{aberrated}(x,y)$ is the aberrated intensity measured at defocus distance $\&$,

A_{mn} are the aberration coefficients which characterize the imperfect imaging system.

If a filter is defined:

$$(19) \quad \tilde{\Omega}(k_x, k_y, \alpha, A_1, A_2, A_3, \ldots) =$$

$$\frac{1}{\sqrt{-2\pi \, \delta z \, \bar{\lambda} (k_x^2 + k_y^2) - 2\sum_m \sum_n A_{mn} k_x^m k_y^n}}$$

Then (18) becomes:

$$(20) \quad \phi(x,y) = F^{-1}\tilde{\Omega}F \frac{1}{I_0} F^{-1}\tilde{\Omega}F\{I_{aberrated}(x,y) - 1\}$$

The term $\{I_{aberrated}(x,y) - 1\}$ is a measure of rate of change of intensity. I_0 intensity is a measurable constant for uniform intensity so that (20) is the same general form as (15). Consequently the special case of aberration can be dealt with by changing the filter in the general method described above. The x and y component filters Ω_x and Ω_y are given by

$$(21) \quad \Omega_x = \Omega_y = \frac{1}{\sqrt{2}} \tilde{\Omega}$$

Figure 3 schematically shows an arrangement for quantitative phase amplitude microscopy. A sample is illuminated using a source of white light Köhler illumination 15, commonly found on optical microscopes. The light is transmitted through an object 16 and collected by a microscope imaging system 17 and relayed to a CCD camera 18 or other digital imaging device having a planar imaging surface. Three images are collected: an in-focus image, I_0, and two slightly out of focus images I_+ and I_-. The defocus is obtained by suitable means such as a

drive system 19 to adjust the microscope focus knob. The
defocus introduced is usually quite small so that
degradation in spatial resolution is minimised, although
the optimal amount of defocus to use is determined by

5   sample properties and imaging geometry such as
magnification, numerical apertures, etc.

When taking the images the numerical aperture of the
condenser is chosen to be less than the numerical aperture

10   of the objective being used. If this is not the case then
serious image degradation will occur, although the precise
amount by which the condenser and objective numerical
apertures should differ involves a tradeoff between image
fidelity and spatial resolution, with the optimal

15   difference depending on the sample properties and the
optics used.

Intensity data from the collected images I_+ and I_- are
subtracted to produce a representative measure of rate of

20   change of intensity (intensity derivative). From this and
the intensity data of collected image I_0 the method
described above can be used to produce quantitative
information about the magnitude of the phase shift in the
image plane.

25

There may be cases in which it is desirable to take more
than two images in order to obtain a better estimate of
the intensity derivative dI/dz. A function can then be
fitted to this data from which dI/dz can be computed and

30   used in the phase determination method in place of the
simple subtraction of two images normally used.

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It is also possible to operate this system in a reflection
geometry to obtain surface topography. The principle of
operation is the same, however the optics have to be
folded back on themselves to form a reflection geometry -
5   otherwise the process is identical.

For certain applications it can also be desirable to
filter the light to a particular wavelength, although this
is not necessary for the described imaging process as it
10   works equally well with white light.

An implementation is shown in Figure 4. An Olympus BX-60
optical microscope 20 was equipped with a set of UMPlan
metallurgical objectives and a universal condenser to
15   provide Köhler illumination. In order to be able to
compare the results with existing imaging modes Nomarski
DIC optics and a set of cover-slip corrected UplanApo
objectives were also acquired for this microscope,
enabling images to be taken of the same field of view
20   using both phase retrieval and Nomarski DIC for the
purposes of qualitative comparison. A 12-bit scientific
grade Photometrics SenSys CCD camera 21 equipped with a
1300x1035 pixel Kodak KAF-1400 CCD chip was added to the
0.5x video port on the microscope to acquire digital
25   images of the sample.

The phase recovery technique of this embodiment of the
invention requires the acquisition of defocused images. A
stepper motor drive system 22 was attached to the focus
30   knob of the microscope. This stepper motor 22 was
attached to the parallel port of a 133MHz Pentium PC 23
also used to control the CCD camera 21, enabling full
automation of the acquisition of through-focus image

sequences. This data acquisition system was linked to
custom software written to recover phase images from the
CCD images, thereby enabling full automation of the image
acquisition and data processing sequences.

5

The form of the differential operators used in the
preferred embodiment of this invention are based on the
optics of the system used to obtain the above-mentioned
images. Thus, the operator takes into account the details
10   of the optical system used to take the images. This is
achieved by:

Determine the numerical aperture of the condenser, $NA_{condenser}$

15   Determine NA of objective, $NA_{objective}$, and $\rho_{objective}$, the radius
of the objective aperture

$$\xi = \frac{NA_{condenser}}{NA_{objective}}$$

20   (These are settings and dimensions on the microscope.)

Determine radial lateral spatial frequency, ρ, and
longitudinal spatial frequency, η.

25   (These are dependent on the pixelation and position
distribution of images taken in the series.)

Determine the wavelength, λ, of the radiation to be used.

30   The new form of the operator is

$$\frac{\sqrt{T_p}}{T_p + \alpha^2}$$

where,

35

$$T_P(\rho) = 2\pi k \int \eta T_P^{(2)}(\rho,\eta)d\eta$$

and

$$T_P^{(2)}(\rho,\eta) = \frac{1}{2\pi\rho}\left\{ \begin{array}{l} \left[\frac{1}{4}\rho_{xy}^2(\xi^2+1)-\frac{1}{4}\rho^2-\left(\frac{\eta}{\lambda\rho}\right)^2-\left|\frac{\eta}{\lambda}-\frac{1}{4}\rho_{xy}^2(\xi^2-1)\right|\right]^{1/2} \\ -\left[\frac{1}{4}\rho_{xy}^2(\xi^2+1)-\frac{1}{4}\rho^2-\left(\frac{\eta}{\lambda\rho}\right)^2-\left|\frac{\eta}{\lambda}+\frac{1}{4}\rho_{xy}^2(\xi^2-1)\right|\right]^{1/2} \end{array} \right\}$$

Figure 5 is a flow chart generally illustrating how T_ρ is determined by means of the above equation merely showing breakdown of the various components of the equation.

Since modifications within the spirit and scope of the invention may readily be effected by persons skilled within the art, it is to be understood that this invention is not limited to the particular embodiment described by way of example hereinabove.

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Claims

1.   A method of quantitative determination of a phase
of a radiation wavefield including the steps of:

5   (a)   producing a representative measure of the
rate of change of intensity of said radiation wave field
over a selected surface extending generally across the
wave field;

             (b)   producing a representative measure of
10   intensity of said radiation wave field over said selected
surface;

             (c)   transforming said measure of rate of change
of intensity to produce a first integral transform
representation and applying to said first integral
15   transform representation a first filter corresponding to
the inversion of a first differential operator reflected
in said measure of rate of change of intensity to produce
a first modified integral transform representation;

             (d)   applying an inverse of said first integral
20   transform to said first modified integral transform
representation to produce an untransformed representation;

             (e)   applying a correction based on said measure
of intensity over said selected surface to said
untransformed representation;

25   (f)   transforming the corrected untransformed
representation to produce a second integral transform
representation and applying to said second integral
transform representation a second filter corresponding to
the inversion of a second differential operator reflected
30   in the corrected untransformed representation to produce a
second modified integral transform representation;

             (g)   applying an inverse of said second integral
transform to said second modified integral transform
representation to produce a measure of phase of said
35   radiation wave field across said selected plane; and

             (h)   wherein at least one of the first or second
differential operator has a form based on an optical

system used to acquire the radiation for producing the representative measure of the rate of change of intensity of the radiation wavefield over the selected surface extending generally across the wavefield.

2.   The method of claim 1 wherein both the first and second differential operators have a form based on the optical system.

3.   The method of claim 2 wherein the first and second integral transforms are produced using a Fourier transform.

4.   The method of claim 2 wherein the differential operators have the form:

$$\frac{\sqrt{T_p}}{T_p + \alpha^2}$$

where,

$$T_p(\rho) = 2\pi\delta\!z \int \eta T_p^{(3)}(\rho,\eta)d\eta$$

and

$$T_p^{(3)}(\rho,\eta) = \frac{i}{2\pi\eta\rho}\left\{ \left[\tfrac{1}{4}\rho_{obj}^2\!\left(\xi^2+1\right)-\tfrac{1}{4}\rho^2-\left(\tfrac{\eta}{\lambda\rho}\right)^2-\left|\tfrac{\eta}{\lambda}-\tfrac{1}{4}\rho_{obj}^2\!\left(\xi^2-1\right)\right|\right]^{\!1/2} \\ -\left[\tfrac{1}{4}\rho_{obj}^2\!\left(\xi^2+1\right)-\tfrac{1}{4}\rho^2-\left(\tfrac{\eta}{\lambda\rho}\right)^2-\left|\tfrac{\eta}{\lambda}+\tfrac{1}{4}\rho_{obj}^2\!\left(\xi^2-1\right)\right|\right]^{\!1/2} \right\}$$

and wherein ρ is radial lateral spatial frequency, the η longitudinal spatial frequency and $\delta\!z$ is the defocus distance in the plane of the object, and

$$\xi = \frac{NA_{condenser}}{NA_{objective}}$$

where $NA_{condenser}$ and $NA_{objective}$ are respectively the numerical aperture of the condensor and the objective, and ρ_{obj} is the maximum spatial frequency accepted by the objective.

5.   A computer program for quantitative determination of the phase of a radiation wavefield including code to perform the method steps according to claim 1.

6.   An apparatus for phase amplitude imaging of an object including:

a radiation wavefield source to irradiate the object;

an imaging system to focus radiation from the object to an imaging surface extending across the wavefield propagating from the object;

means to produce a representative measure of radiation intensity over the imaging surface; and

processing means to:

(i)   produce a representative measure of intensity and a representative measure of rate of change of intensity in the direction of radiation propagation over a selected surface extending across the wave field from representative measures of radiation intensity over said image surface at said first focus and said second focus;

(ii)   transform said measure of rate of change of intensity to produce a first integral transform representation;

(iii)   apply to said first integral transform representation a first filter corresponding to the inversion of a first differential operator reflected in said measure of rate of change of intensity to produce a first modified integral transform representation;

(iv)   apply an inverse of said first integral transform to said first modified integral transform representation to produce an untransformed representation;

(v)   apply a correction based on said measure

of intensity over said selected surface to said untransformed representation;

  (vi) transform the corrected untransformed representation to produce a second integral transform representation;

  (vii) apply to said second integral transform representation a second filter corresponding to the inversion of a second differential operator reflected in the corrected untransformed representation to produce a second modified integral transform representation; and

  (viii) apply an inverse of said second integral transform to said second modified integral transform representation to produce a measure of phase of said radiation wave field across said selected plane; and

  (ix) wherein at least one of the first or second differential operators has a form based on the imaging system to focus the radiation from the object to the imaging surface.

7. The apparatus of claim 6 wherein the differential operator, in Fourier space, is

$$\frac{\sqrt{T_p}}{T_p + \alpha^2}$$

where,

$$T_p(\rho) = 2\pi i \& \int \eta T_p^{(0)}(\rho, \eta) d\eta$$

and

$$T_p^{(0)}(\rho, \eta) = \frac{i}{2\pi \rho} \left[\left[\tfrac{1}{4}\rho_{obj}^2(\xi^2 + 1) - \tfrac{1}{4}\rho^2 - \left(\frac{\eta}{\lambda\rho}\right)^2 - \left|\frac{\eta}{\lambda} - \tfrac{1}{4}\rho_{obj}^2(\xi^2 - 1)\right|\right]^{\frac{1}{2}} - \left[\tfrac{1}{4}\rho_{obj}^2(\xi^2 + 1) - \tfrac{1}{4}\rho^2 - \left(\frac{\eta}{\lambda\rho}\right)^2 - \left|\frac{\eta}{\lambda} + \tfrac{1}{4}\rho_{obj}^2(\xi^2 - 1)\right|\right]^{\frac{1}{2}} \right]$$

and wherein ρ is radial lateral spatial frequency, the η longitudinal spatial frequency and $\&$ is the defocus distance in the plane of the object, and

5
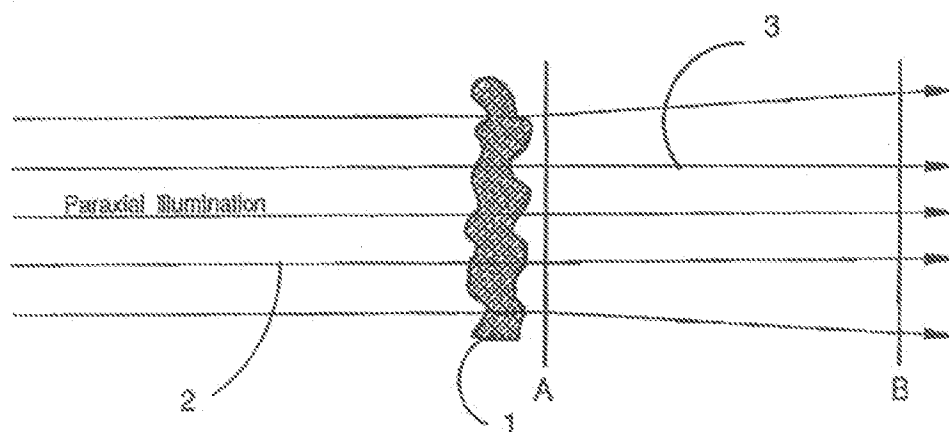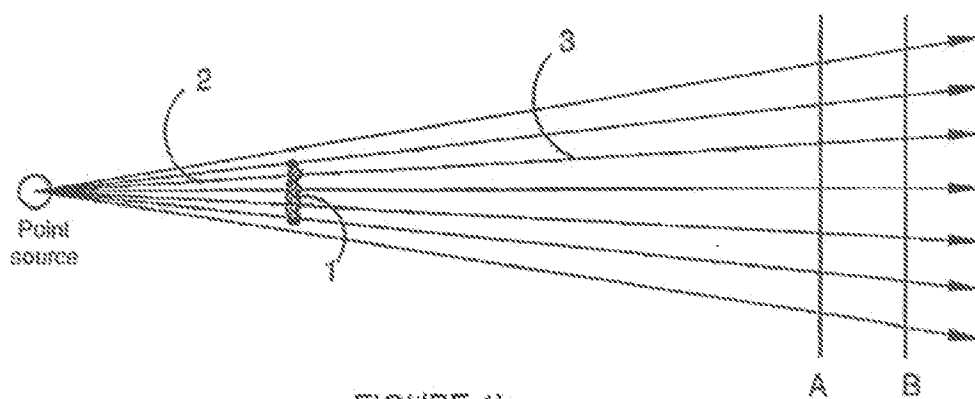
$$\xi = \frac{NA_{condensor}}{NA_{objective}}$$

where $NA_{condensor}$ and $NA_{objective}$ are respectively the numerical aperture of the condensor and the objective, and ρ_{obj} is the maximum spatial frequency accepted by the objective.

10

8.   The apparatus of claim 7 wherein both the first and second differential operators have the above form.
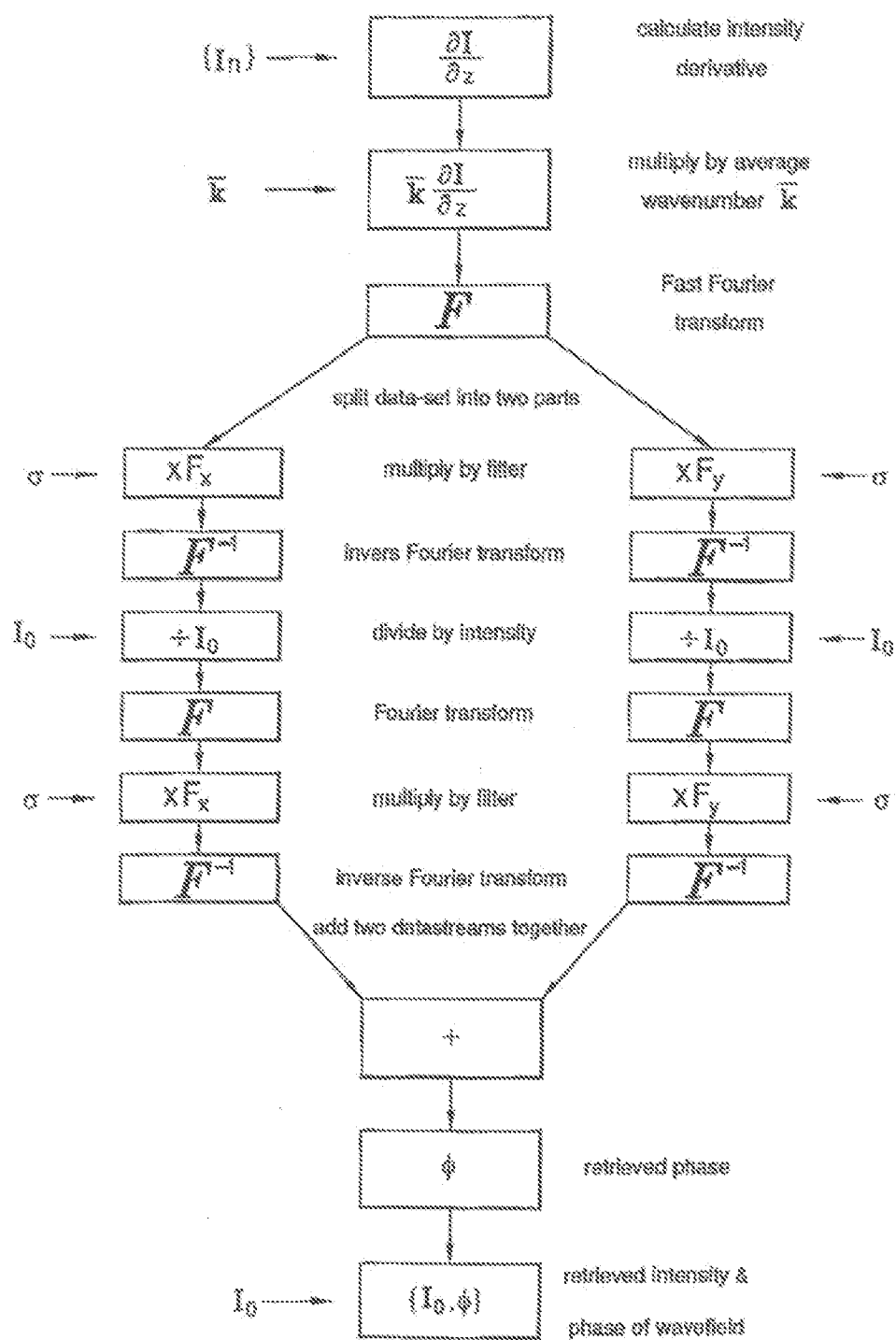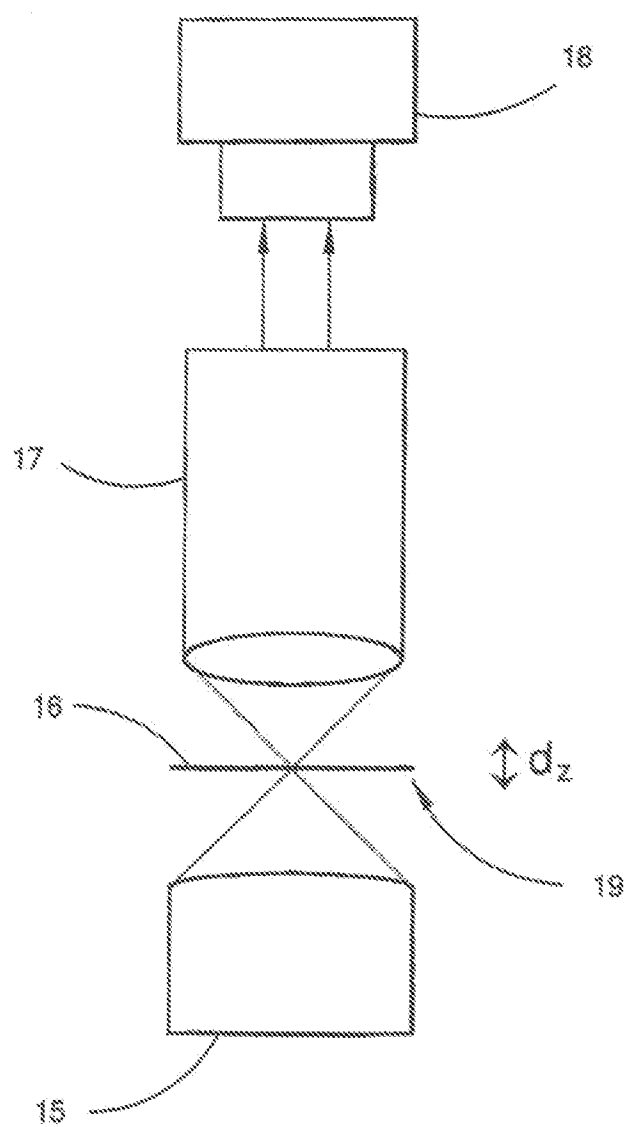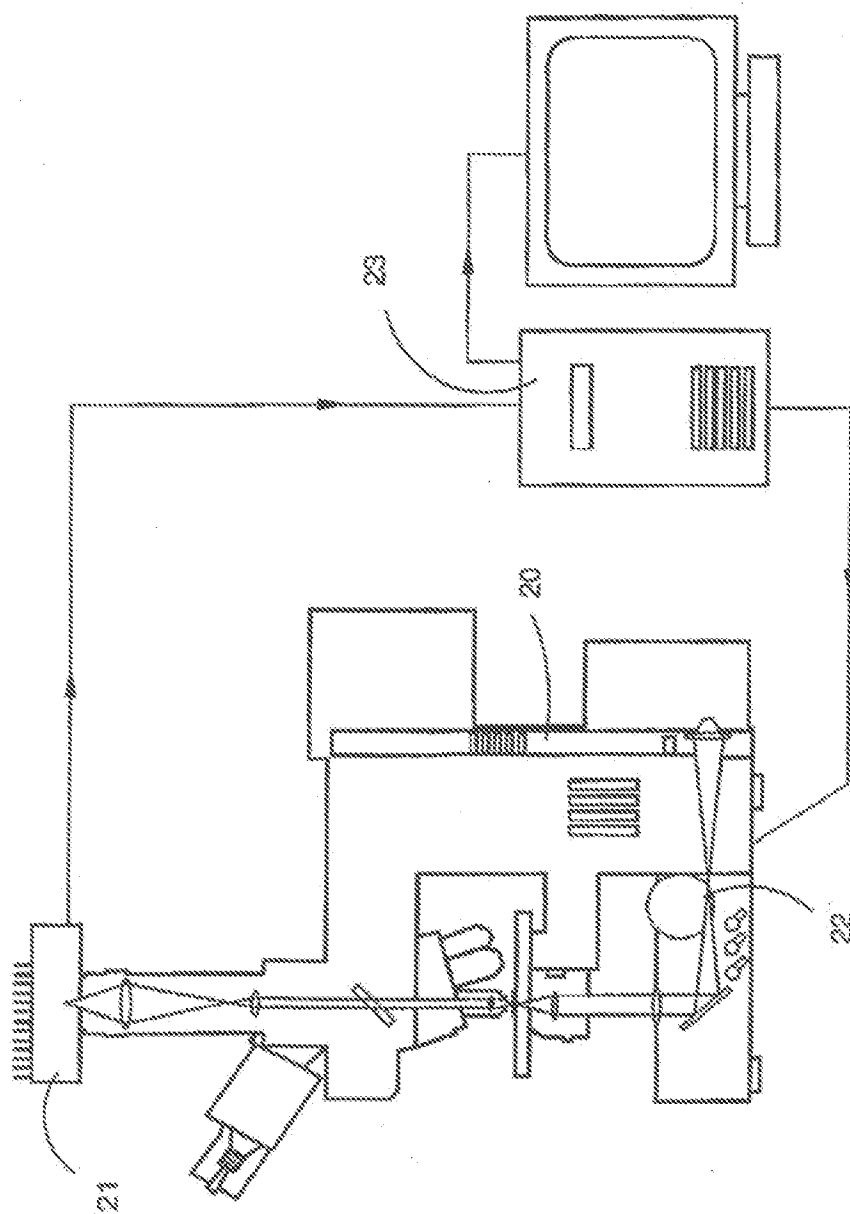
FIGURE 1a

FIGURE 1b

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FIGURE 2

FIGURE 3
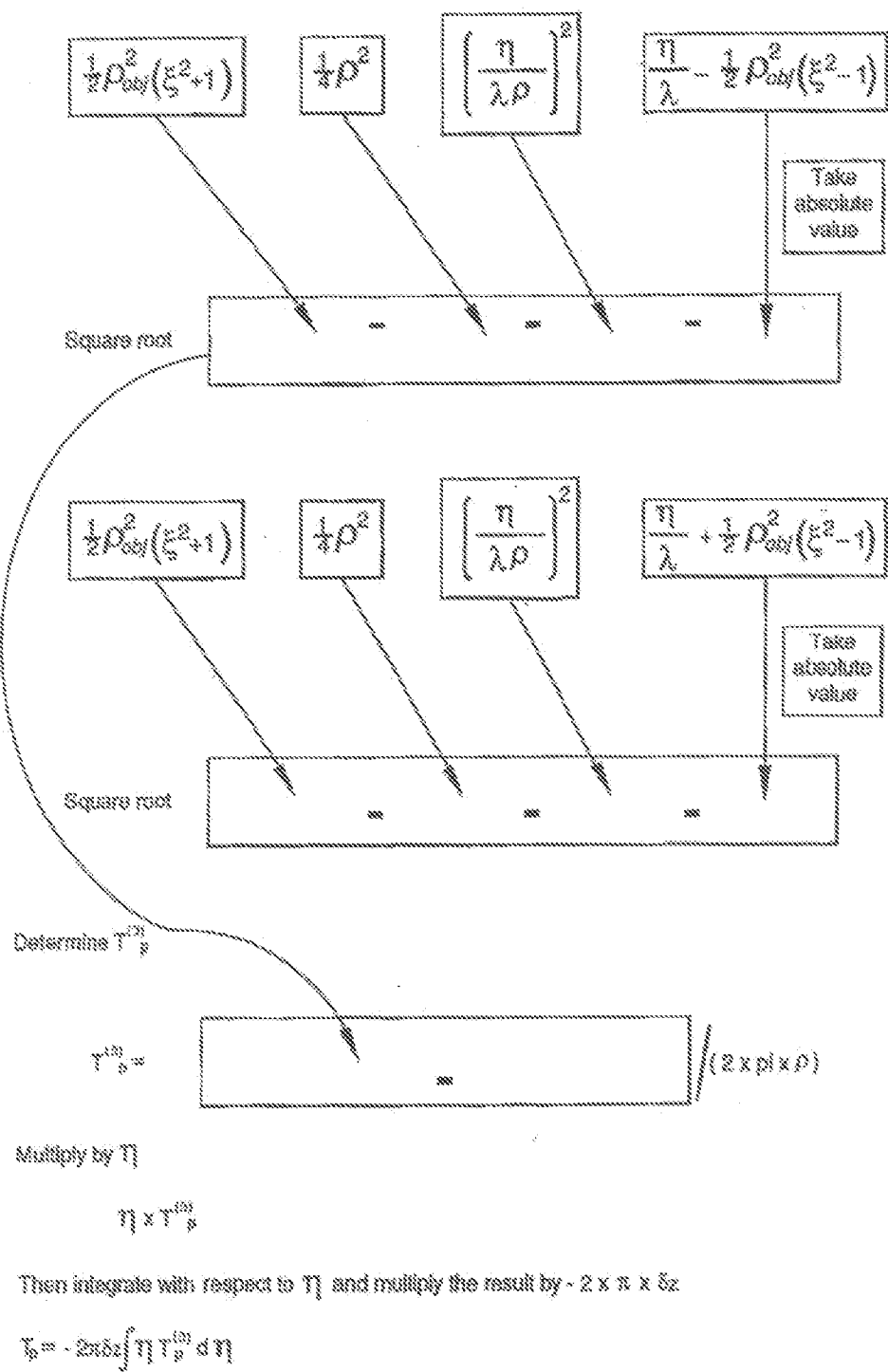
FIGURE 4

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$$\frac{1}{2}\rho_{obj}^2(\xi^2+1)$$

$$\frac{1}{2}\rho^2$$

$$\left[\frac{\eta}{\lambda\rho}\right]^2$$

$$\frac{\eta}{\lambda} - \frac{1}{2}\rho_{obj}^2(\xi^2-1)$$

Take absolute value

Square root

$$\frac{1}{2}\rho_{obj}^2(\xi^2+1)$$

$$\frac{1}{2}\rho^2$$

$$\left[\frac{\eta}{\lambda\rho}\right]^2$$

$$\frac{\eta}{\lambda} + \frac{1}{2}\rho_{obj}^2(\xi^2-1)$$

Take absolute value

Square root

Determine $T_p^{(3)}$

$$T_p^{(3)} = \boxed{\qquad\qquad\cdots\qquad\qquad} / (2 \times pi \times \rho)$$

Multiply by η

$$\eta \times T_p^{(3)}$$

Then integrate with respect to η and multiply the result by $-2 \times \pi \times \delta_z$

$$\tau_b = -2\pi\delta_z \int \eta \, T_p^{(3)} \, d\eta$$

FIGURE 5

A.	CLASSIFICATION OF SUBJECT MATTER

Int. Cl. [7]:   G01J 9/00

According to International Patent Classification (IPC) or to both national classification and IPC

B.	FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

DWPI, JAPIO   Keywords: phase; intensit, irradiance; wavefront, wavefield; recover, retriev, profil, reconstruct, image; transport, derivative, differential, change, rate; G01J 9/00; transform, operator, comput, calculat, solv, solution; fourier, fft

C.	DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	WO 00/26622 A (THE UNIVERSITY OF MELBOURNE) 11 May 2000 Pages 23-24, Figure 2	1-3, 5-6
A	US 5633714 A (NYYSSONEN) 27 May 1997 Whole document	1-8

☐ Further documents are listed in the continuation of Box C   ☒ See patent family annex

*	Special categories of cited documents:		
"A"	document defining the general state of the art which is not considered to be of particular relevance	"T"	later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention
"E"	earlier application or patent but published on or after the international filing date	"X"	document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
"L"	document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)	"Y"	document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art
"O"	document referring to an oral disclosure, use, exhibition or other means	"&"	document member of the same patent family
"P"	document published prior to the international filing date but later than the priority date claimed		

Date of the actual completion of the international search	Date of mailing of the international search report
29 November 2002	0 6 DEC 2002

Name and mailing address of the ISA/AU	Authorized officer
AUSTRALIAN PATENT OFFICE PO BOX 200, WODEN ACT 2606, AUSTRALIA E-mail address: pct@ipaustralia.gov.au Facsimile No. (02) 6283 3929	MICHAEL HALL Telephone No : (02) 6283 2474

This Annex lists the known "A" publication level patent family members relating to the patent documents cited in the above-mentioned international search report. The Australian Patent Office is in no way liable for these particulars which are merely given for the purpose of information.

Patent Document Cited in Search Report		Patent Family Member					
WO	200026622	AU	200015005	BR	9914976	EP	1127252
US	5633714	NONE					

END OF ANNEX